Computer Controlled Systems

supplementary test – Jan. 11, 2018

(The answers can be given in Hungarian)

Theoretical questions (25 points)

1. Define the convolution in the case of continuous signals. (5p)
2. Define the Dirac-δ function. How can we compute its Laplacian? (5p)
3. What is the Bode-diagram of a $H(s)$ transfer function? What kind of information can be obtain by means of that? (5p)
4. Describe the problem statement of the LQR control design. (i.e., what are the known data, what is the objective function to be optimized and what is to be computed?). What kind of feedback is obtained? (5p)
5. Describe the problem statement of the Kalman filter. What are the assumptions on the different signals? (5p)

Computational exercises (25 points)

1. Consider the following model

$$\frac{dx_1}{dt} = x_2$$
$$\frac{dx_2}{dt} = 3x_1$$

   where $x_1,\ x_2 \in \mathbb{R}$: Check whether or not $V(x) = x_1^2 + x_2^2$ is an appropriate Lyapunov function for the above system. (5p)

2. Consider the following state space model

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 1 \end{bmatrix}.$$

   (a) Design a pole-placement controller such that the poles of the closed loop system are at $[-1\ \ -1]$. (3p)

   (b) Check your results. (2p)

3. Consider the following state space model

$$A = \begin{bmatrix} 4 & 0 \\ 3 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 \end{bmatrix}.$$

   Determine the matrices Φ and Γ of the corresponding discrete time system with sampling time $h = ln(4)$. (5p)

4. Given the state equation:

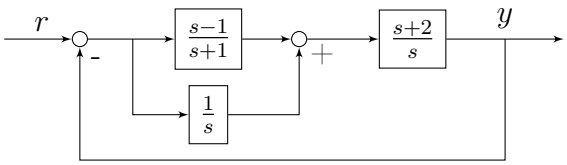
$$\dot{x} = -0.5x + 2u$$

   Determine the input signal – in the form of a state feedback – minimizing the cost function

$$J = \frac{1}{2}\int_0^\infty x^T Q x + u^T R u \ dt = \frac{1}{2}\int_0^\infty Q x^2 + R u^2 \ dt$$

   (5p)

5. Determine the overall transfer function of the following block diagram (4p)



Is the resulting transfer fuction strictly proper? (1p)